For the memories of yesterday.
For the happiness of today...

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دوال يجب حفظها

1. $A \, rect\left(\dfrac{t}{\tau}\right) \rightleftharpoons A\tau \, sinc(f\tau)$

2. $e^{-t} \cdot u(t) \rightleftharpoons \dfrac{1}{1 + J2\pi f}$ as 3 but $\alpha = 1$

3. $e^{-\alpha t} \cdot u(t) \rightleftharpoons \dfrac{1}{\alpha + J2\pi f}$

4. $e^{\alpha t} \cdot u(-t) \rightleftharpoons \dfrac{1}{\alpha - J2\pi f}$

5. $e^{t} \cdot u(-t) \rightleftharpoons \dfrac{1}{1 - J2\pi f}$ as 4 but $\alpha = 1$

6. $sgn(t) \rightleftharpoons \dfrac{1}{J\pi f}$

7. $u(t) \rightleftharpoons \dfrac{1}{2}\left[\dfrac{1}{J\pi f} + \delta(f)\right]$

8. $A \rightleftharpoons A \cdot \delta(f)$

9. $\delta(t) \rightleftharpoons 1$

10. $m(t) \cdot \cos(2\pi f_c t) \rightleftharpoons \dfrac{1}{2}\left[M(f - f_c) + M(f + f_c)\right]$ freq. shift

11. $A_c \cdot \cos(2\pi f_c t) \rightleftharpoons \dfrac{A_c}{2}\left[\delta(f - f_c) + \delta(f + f_c)\right]$ freq. shift

12. $A\tau \, sinc(t\tau) \rightleftharpoons A \, rect\left(\dfrac{f}{\tau}\right)$ duality with 1

13. $1 \cdot e^{-J2\pi f_0 t} \rightleftharpoons \delta(f + f_0)$ freq. shift

When you're down and troubled
And you need some love care

إثباتات الخواص

$$\star \; G(f) = \int_{-\infty}^{\infty} g(t) \; e^{-j2\pi f t} \; dt \qquad F.T.$$

$$\star \; g(t) = \int_{-\infty}^{\infty} G(f) \cdot e^{+j2\pi f t} \cdot df \qquad I.F.T.$$

① Linearity

$$\int_{-\infty}^{\infty} \left[a \cdot g_1(t) + b \cdot g_2(t) \right] e^{-j2\pi f t} \; dt$$

$$= \int_{-\infty}^{\infty} a \cdot g_1(t) \; e^{-j2\pi f t} \; dt \; + \; b \int_{-\infty}^{\infty} g_2(t) \cdot e^{-j2\pi f t} \; dt$$

$$\underbrace{\qquad\qquad}_{g_2(t) \; F.T. \; مطلوب}$$

$$= a \cdot G_1(f) \qquad + \quad b \cdot G_2(f)$$

② Time Scaling

$$F[g(at)] = \int_{-\infty}^{\infty} g(at) \cdot e^{-j2\pi f t} \; dt \qquad\longrightarrow \boxed{1}$$

$$\text{let } \; \tau = at \; \longrightarrow \; d\tau = a \cdot dt$$
$$t = \tau/a \qquad dt = \frac{d\tau}{a}$$
$$عوض \; في \; \boxed{1}$$

$$F[g(at)] = \frac{1}{a} \int_{-\infty}^{\infty} g(\tau) \cdot e^{-j2\pi \frac{f}{a} \tau} \cdot d\tau$$

$$= \frac{1}{a} \cdot G(f/a)$$

$$\boxed{1}$$

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③ Duality

$$g(t) = \int_{-\infty}^{\infty} G(f) \cdot e^{+j2\pi ft} \, df$$

replace t by $-t$ in both sides

$$g(-t) = \int_{-\infty}^{\infty} G(f) \cdot e^{-j2\pi ft} \, df$$

replace t by f and f by t

$$\therefore g(-f) = \int_{-\infty}^{\infty} G(t) \cdot e^{-j2\pi ft} \, dt$$

$$\therefore G(t) \rightleftharpoons g(-f)$$

④ Time Shift

$$F[g(t-t_0)] = \int_{-\infty}^{\infty} g(t-t_0) \cdot e^{-j2\pi ft} \, dt \rightarrow \text{ت-to ولي}$$

ولي
$t-t_0$

let $\tau = t-t_0 \rightarrow d\tau = dt$
$$t = \tau + t_0$$

$$= \int_{-\infty}^{\infty} g(\tau) \cdot e^{-j2\pi f(\tau + t_0)} \, d\tau$$

$$= \int_{-\infty}^{\infty} g(\tau) \cdot e^{-j2\pi f\tau} \cdot e^{-j2\pi f t_0} \, d\tau$$

ثابت يطلع
برا التكامل

$$= e^{-j2\pi f t_0} \cdot \int_{-\infty}^{\infty} g(\tau) \cdot e^{-j2\pi f\tau} \, d\tau$$

$$= e^{-j2\pi f t_0} \cdot \underbrace{G(f)}$$

⑤ Frequency Shift

$$F[g(t) \cdot e^{-j2\pi f_0 t}] = \int_{-\infty}^{\infty} g(t) \cdot e^{-j2\pi f_0 t} \cdot e^{-j2\pi f t} dt$$

$$= \int_{-\infty}^{\infty} g(t) \cdot e^{-j2\pi (f+f_0)t} dt$$

$$= G(f + f_0)$$

⑥ Area under $g(t)$

$$Area = \int_{-\infty}^{\infty} g(t) \cdot dt = \text{Fourier Transform eq. at } f=0$$

$$= \int_{-\infty}^{\infty} g(t) \cdot \underbrace{e^{-j2\pi f t}}_{\text{at } f=0} dt$$

$$= G(0)$$

⑦ Area under $G(f)$

$$Area = \int_{-\infty}^{\infty} G(f) \, df = \text{inverse Fourier eq. at } t=0$$

$$= \int_{-\infty}^{\infty} G(f) \cdot \underbrace{e^{+j2\pi f t}}_{\text{at } t=0} df$$

$$= g(0)$$

⑧ Differentiation in time domain

$$\because \underline{g(t)} = \int_{-\infty}^{\infty} \underline{G(f)} \cdot e^{j2\pi f t} df$$

$$\text{and} \quad g(t) \rightleftharpoons G(f)$$

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$$\frac{d}{dt} g(t) = \int_{-\infty}^{\infty} (j 2\pi f) \cdot G(f) \cdot e^{+j2\pi f t} df$$

$$\therefore \quad \frac{d}{dt} g(t) \rightleftharpoons (j 2\pi f) \cdot G(f)$$

⑨ Integeration in time   & $\frac{d^n}{dt} g(t) \rightleftharpoons (j 2\pi f)^n \, G(f)$

Prove : $\int_{-\infty}^{\infty} g(t) \, dt \xrightarrow{\text{F.T.}} \frac{1}{j 2\pi f} G(f)$

$$\because \quad g(t) = \frac{d}{dt} \left[\int_{-\infty}^{\infty} g(t) \, dt \right]$$

$$\downarrow \text{F.T.}$$

from ⑧

$$G(f) = (j 2\pi f) \cdot F\left[\int_{-\infty}^{\infty} g(t) \, dt \right]$$

$$\therefore \quad F\left[\int_{-\infty}^{\infty} g(t) \, dt \right] = \frac{G(f)}{(j 2\pi f)}$$

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